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Testing process capability for one-sided specification limit with application to the voltage level translator

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Abstract

Process capability indices C_{PU} and C_{PL} have been widely used in the microelectronics manufacturing industry as capability measures for processes with one-sided specification limits. In this paper, the theory of statistical hypothesis testing is implemented for normal processes, using the uniformly minimum variance unbiased estimators of C_{PU} and C_{PL}. Efficient SAS computer programs are provided to calculate the critical values and the *p*-values required for making decisions. Useful critical values for some commonly used capability requirements are tabulated. Based on the test a simple but practical step-by-step procedure is developed for in-plant applications. An example on the voltage level translator manufacturing process is given to illustrate how the proposed procedure may be applied to test whether the process meets the preset capability requirement.

1. Introduction

Several capability indices including C_p, C_{PU}, C_{PL}, and C_{pk} have been widely used in the manufacturing industry as well as the service industry providing common quantitative measures on process potential and performance (see Kane [1], Pearn et al. [2], and Pearn and Chen [3,4]). Those indices are defined in the following:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma},$$

$$C_{PU} = \frac{\text{USL} - \mu}{3\sigma},$$

$$C_{PL} = \frac{\mu - \text{LSL}}{3\sigma},$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation (overall process variation). While C_p and C_{pk} are appropriate measures for processes with two-sided specifications (which require both USL and LSL), C_{PU} and C_{PL} have been designed specifically for processes with one-sided specification limit (which require only USL or LSL). The index C_{PU} measures the capability of a smaller-the-better process with an upper specification limit USL, whereas the index C_{PL} measures the capability of a larger-the-better process with a lower specification limit LSL.

For normally distributed processes with one-sided specification limit USL, the process yield is

$$P(X < \text{USL}) = P\left(\frac{X-\mu}{3\sigma} < \frac{\text{USL}-\mu}{3\sigma}\right) = P\left(\frac{1}{3}Z < C_{PU}\right) = P(Z < 3C_{PU}) = \Phi(3C_{PU}),$$

where Z follows the standard normal distribution $N(0,1)$ with the cumulated distribution function $\Phi(\cdot)$. Similarly, for normally distributed processes with one-sided specification limit LSL, the process yield is

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Table 1
C_{PU} and the corresponding non-conforming units (in ppm)

Values of C_{PU}	NCPPM
0.80	8198
1.00	1350
1.25	88
1.45	7
1.60	1
1.80	0.33
2.00	0.00099

$$P(X > \mathrm{LSL}) = P\left(\frac{\mu - X}{3\sigma} < \frac{\mu - \mathrm{LSL}}{3\sigma}\right) = P\left(-\frac{1}{3}Z < C_{PL}\right) = P(Z > -3C_{PL}) = 1 - \Phi(-3C_{PL}) = \Phi(3C_{PL}).$$

Table 1 displays the corresponding non-conforming units in parts per million (NCPPM) for a well controlled normally distributed process.

Montgomery [5] recommended some minimum quality requirements of C_{PU} and C_{PL} for processes runs under some designated capable conditions. In particular, 1.25 for existing processes, and 1.45 for new processes; 1.45 also for existing processes on safety, strength, or critical parameter, and 1.60 for new processes on safety, strength, or critical parameter.

The formulae for these indices are easy to understand and straightforward to apply. In practice, however, sample data must be collected in order to calculate these indices. Therefore, a great degree of uncertainty may be introduced into capability assessments due to sampling errors. To ensure the capability assessment reliable, the uniformly minimum variance unbiased estimators (UMVUEs) of C_{PU} and C_{PL} obtained by Pearn and Chen [6], is considered under normality assumption. The theory of testing hypothesis using the UMVUEs of C_{PU} and C_{PL} is implemented, and efficient SAS computer programs are provided to calculate the critical values and the p-values required for making decisions.

Based on the test, two practical step-by-step procedures are developed for in-plant applications. The practitioners or the engineers can apply the proposed procedure to judge whether or not their processes meet the preset capability requirement and run under the desired quality conditions. An example taken from the factory involving the voltage level translator manufacturing is investigated. The proposed procedures are applied to test whether the process meets the preset capability requirement. If the process meets the capability requirement, the translators made are considered to be reliable.

2. Estimations of C_{PU} and C_{PL}

To estimate the indices C_{PU} and C_{PL}, Chou and Owen [7] considered $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$, the natural estimators of C_{PU} and C_{PL}, which are defined as the following:

$$\widehat{C}_{PU} = \frac{\mathrm{USL} - \overline{X}}{3S}, \quad \widehat{C}_{PL} = \frac{\overline{X} - \mathrm{LSL}}{3S},$$

where $\overline{X} = \sum_{i=1}^{n} X_i/n$ is the sample mean, and $S^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is the sample variance, which may be obtained from a process that is demonstrably stable (under statistical control).

Chou and Owen [7] showed that under normality assumption, the estimator $\widehat{C}_{PU}$ is distributed as $C_n t_{n-1}(\delta)$, where $C_n = (3\sqrt{n})^{-1}$, and $t_{n-1}(\delta)$ is a non-central t distribution with $n-1$ degrees of freedom and non-centrality parameter $\delta = 3\sqrt{n}C_{PU}$. The estimator $\widehat{C}_{PL}$ has the same sampling distribution as $\widehat{C}_{PU}$ but with $\delta = 3\sqrt{n}C_{PL}$.

Both estimators $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$ are biased. But, Pearn and Chen [6] showed that by adding the correction factor $b_{n-1} = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ to $\widehat{C}_{PU}$ and $\widehat{C}_{PL}$, we may obtain unbiased estimators $b_{n-1}\widehat{C}_{PU}$ and $b_{n-1}\widehat{C}_{PL}$ which have been denoted as $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$. Thus, we have $E(\widetilde{C}_{PU}) = C_{PU}$, and $E(\widetilde{C}_{PL}) = C_{PL}$. Since $b_{n-1} < 1$, then $\mathrm{var}(\widetilde{C}_{PU}) < \mathrm{var}(\widehat{C}_{PU})$ and $\mathrm{var}(\widetilde{C}_{PL}) < \mathrm{var}(\widehat{C}_{PL})$.

Pearn and Chen [6] further showed that $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$ are the UMVUEs of C_{PU} and C_{PL}, respectively. Therefore, in practice using the UMVUEs $\widetilde{C}_{PU}$ and $\widetilde{C}_{PL}$ to calculate the capability measures would be desirable. The probability density function (PDF) of $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$) may be easily obtained as

$$f(x) = \frac{3\sqrt{n/(n-1)}2^{-n/2}}{b_{n-1}\sqrt{\pi}\Gamma[(n-1)/2]} \times \int_0^\infty y^{(n-2)/2} \times \exp\left\{-\frac{1}{2}\left[y + \left(\frac{3x\sqrt{ny}}{b_{n-1}\sqrt{n-1}} - \delta\right)^2\right]\right\}\mathrm{d}y,$$

where $b_{n-1} = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ and $\delta = 3\sqrt{n}C_{PU}$ (or $\delta = 3\sqrt{n}C_{PL}$). Figs. 1–4 display the PDF plots of $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$) for C_{PU} (or C_{PL}) = 1.00, 1.25, 1.45, and 1.60, with various sample sizes of n = 10, 20, 30, 40, and 50 (from bottom to top in plot).

3. Testing process capability

To test whether a given process with an upper specification limit USL, meets the preset capability requirement and runs under the desired quality condition, the hypothesis testing: H_0: $C_{PU} \leqslant C$ versus H_1: $C_{PU} > C$, is considered, where C is a known constant preset by the product designer or process engineer. A process meets

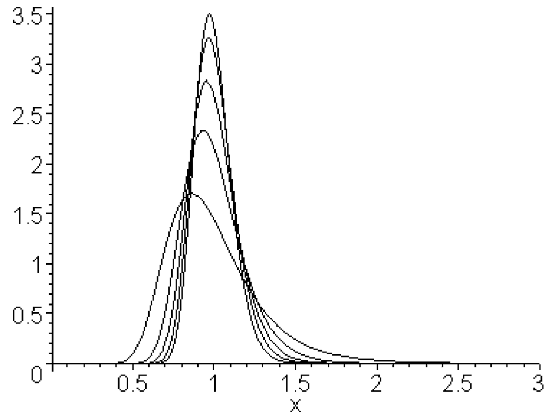

Fig. 1. PDF plot of $\widetilde{C}_{PU}$ for $C_{PU} = 1.00$ and $n = 10, 20, 30, 40, 50$ (from bottom to top in plot).

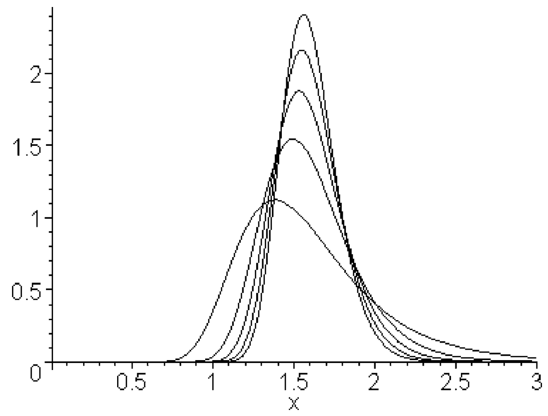

Fig. 4. PDF plot of $\widetilde{C}_{PU}$ for $C_{PU} = 1.60$ and $n = 10, 20, 30, 40, 50$ (from bottom to top in plot).

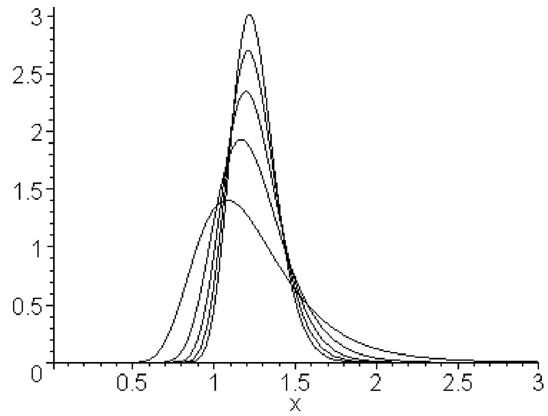

Fig. 2. PDF plot of $\widetilde{C}_{PU}$ for $C_{PU} = 1.25$ and $n = 10, 20, 30, 40, 50$ (from bottom to top in plot).

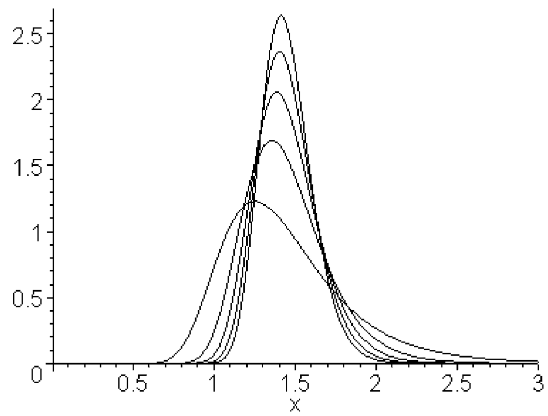

Fig. 3. PDF plot of $\widetilde{C}_{PU}$ for $C_{PU} = 1.45$ and $n = 10, 20, 30, 40, 50$ (from bottom to top in plot).

the capability requirement if $C_{PU} > C$, and fails to meet the capability requirement if $C_{PU} \leqslant C$.

Based on a given $\alpha(c_0) = \alpha$, the chance of incorrectly judging an incapable process as capable, the decision rule is to reject H_0 if $\widetilde{C}_{PU} > c_0$ and do not reject H_0 otherwise, for some critical value $c_0 > C$. It is noted that the procedures for testing the hypothesis: 'H_0: $C_{PL} \leqslant C$ versus H_1: $C_{PL} > C$' and 'H_0: $C_{PU} \leqslant C$ versus H_1: $C_{PU} > C$' are exactly the same.

The p-value, which is the actual probability of incorrectly concluding an incapable process as a capable one, corresponding to a specific value of $\widetilde{C}_{PU} = w$, can be calculated from the sample as

$$P\{\widetilde{C}_{PU} \geqslant w | C_{PU} \leqslant C\} = P\left\{3\sqrt{n}\widehat{C}_{PU} \geqslant \frac{3\sqrt{n}w}{b_{n-1}} | C_{PU} \leqslant C\right\}$$
$$= P\left\{t_{n-1}(\delta) \geqslant \frac{3\sqrt{n}w}{b_{n-1}}\right\},$$

where $\delta = 3\sqrt{n}C$. An efficient SAS computer program is developed to calculate the p-value given a specific value of $\widetilde{C}_{PU} = w$ obtained from the sample data. The program is listed in Appendix A, with input parameters set to: $C = 1.25$, sample size $n = 120$, and the calculated $\widetilde{C}_{PU} = w = 1.433$. The program gives the p-value as 0.025.

On the other hand, the critical value, c_0, with fixed α risk, may be determined by solving the following equations. An efficient SAS computer program (see Appendix B) is developed to calculate the critical values c_0 for a specific value of C.

$$P\left\{\widetilde{C}_{PU} \geqslant c_0 | C_{PU} = C\right\} = \alpha,$$

$$P\left\{3\sqrt{n}\widehat{C}_{PU} \geqslant \frac{3\sqrt{n}c_0}{b_{n-1}} | C_{PU} = C\right\} = \alpha,$$

$$P\left\{t_{n-1}(\delta) \geqslant \frac{3\sqrt{n}c_0}{b_{n-1}}\right\} = \alpha,$$

where $\delta = 3\sqrt{n}C$. Hence, $3\sqrt{n}c_0/b_{n-1} = t_{n-1,\alpha}(\delta)$, the upper αth percentile of $t_{n-1}(\delta)$, the non-central t distribution. Thus, the critical value c_0 may be obtained as $c_0 = b_{n-1}t_{n-1,\alpha}(\delta)/(3\sqrt{n})$, where $\delta = 3\sqrt{n}C$.

Critical values for those capability requirements $C_{PU} = 1.25$, 1.45, 1.60, as recommended by Montgomery [5], are summarized in Tables 2–4 for sample sizes

Table 2
Critical values c_0 for $C = 1.25$, $n = 10(5)505$ and $\alpha = 0.01$, 0.025, 0.05

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	2.420	2.124	1.910	260	1.396	1.371	1.350
15	2.087	1.895	1.750	265	1.395	1.370	1.349
20	1.927	1.780	1.667	270	1.393	1.369	1.348
25	1.830	1.708	1.614	275	1.392	1.368	1.347
30	1.763	1.659	1.576	280	1.391	1.367	1.346
35	1.715	1.622	1.548	285	1.389	1.366	1.346
40	1.677	1.593	1.526	290	1.388	1.365	1.345
45	1.647	1.570	1.508	295	1.387	1.363	1.344
50	1.622	1.551	1.493	300	1.386	1.362	1.343
55	1.602	1.535	1.481	305	1.384	1.362	1.342
60	1.584	1.521	1.470	310	1.383	1.361	1.342
65	1.568	1.509	1.460	315	1.382	1.360	1.341
70	1.555	1.498	1.452	320	1.381	1.359	1.340
75	1.543	1.489	1.444	325	1.380	1.358	1.339
80	1.532	1.480	1.438	330	1.379	1.357	1.339
85	1.522	1.472	1.432	335	1.378	1.356	1.338
90	1.513	1.465	1.426	340	1.377	1.355	1.337
95	1.505	1.459	1.421	345	1.376	1.355	1.337
100	1.498	1.453	1.416	350	1.375	1.354	1.336
105	1.491	1.448	1.412	355	1.374	1.353	1.335
110	1.485	1.443	1.408	360	1.373	1.352	1.335
115	1.479	1.438	1.404	365	1.372	1.351	1.334
120	1.474	1.434	1.401	370	1.371	1.351	1.333
125	1.469	1.430	1.398	375	1.370	1.350	1.333
130	1.464	1.426	1.394	380	1.370	1.349	1.332
135	1.460	1.422	1.392	385	1.369	1.349	1.332
140	1.455	1.419	1.389	390	1.368	1.348	1.331
145	1.451	1.416	1.386	395	1.367	1.347	1.331
150	1.448	1.413	1.384	400	1.366	1.347	1.330
155	1.444	1.410	1.382	405	1.366	1.346	1.330
160	1.441	1.407	1.379	410	1.365	1.345	1.329
165	1.438	1.405	1.377	415	1.364	1.345	1.329
170	1.434	1.402	1.375	420	1.363	1.344	1.328
175	1.432	1.400	1.373	425	1.363	1.344	1.328
180	1.429	1.398	1.372	430	1.362	1.343	1.327
185	1.426	1.395	1.370	435	1.361	1.343	1.327
190	1.424	1.393	1.368	440	1.361	1.342	1.326
195	1.421	1.391	1.367	445	1.360	1.341	1.326
200	1.419	1.389	1.365	450	1.359	1.341	1.325
205	1.417	1.388	1.364	455	1.359	1.340	1.325
210	1.414	1.386	1.362	460	1.358	1.340	1.325
215	1.412	1.384	1.361	465	1.357	1.339	1.324
220	1.410	1.383	1.359	470	1.357	1.339	1.324
225	1.408	1.381	1.358	475	1.356	1.338	1.323
230	1.406	1.379	1.357	480	1.356	1.338	1.323
235	1.405	1.378	1.356	485	1.355	1.337	1.323
240	1.403	1.377	1.355	490	1.354	1.337	1.322
245	1.401	1.375	1.353	495	1.354	1.337	1.322
250	1.400	1.374	1.352	500	1.353	1.336	1.321
255	1.398	1.373	1.351	505	1.353	1.336	1.321

Table 3
Critical values c_0 for $C = 1.45$, $n = 10(5)505$ and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	2.793	2.452	2.206	260	1.617	1.589	1.565
15	2.410	2.189	2.023	265	1.616	1.587	1.563
20	2.225	2.057	1.927	270	1.614	1.586	1.562
25	2.114	1.975	1.866	275	1.612	1.584	1.561
30	2.038	1.918	1.823	280	1.611	1.583	1.560
35	1.982	1.876	1.791	285	1.609	1.582	1.559
40	1.939	1.843	1.766	290	1.608	1.581	1.558
45	1.904	1.816	1.745	295	1.606	1.580	1.557
50	1.876	1.794	1.728	300	1.605	1.578	1.556
55	1.852	1.776	1.714	305	1.603	1.577	1.555
60	1.832	1.760	1.701	310	1.602	1.576	1.554
65	1.814	1.746	1.690	315	1.601	1.575	1.554
70	1.798	1.734	1.681	320	1.600	1.574	1.553
75	1.785	1.723	1.672	325	1.598	1.573	1.552
80	1.772	1.713	1.664	330	1.597	1.572	1.551
85	1.761	1.704	1.657	335	1.596	1.571	1.550
90	1.751	1.696	1.651	340	1.595	1.570	1.550
95	1.742	1.689	1.645	345	1.594	1.569	1.549
100	1.734	1.682	1.640	350	1.593	1.568	1.548
105	1.726	1.676	1.635	355	1.592	1.568	1.547
110	1.719	1.670	1.630	360	1.590	1.567	1.547
115	1.712	1.665	1.626	365	1.589	1.566	1.546
120	1.706	1.660	1.622	370	1.588	1.565	1.545
125	1.700	1.655	1.619	375	1.587	1.564	1.545
130	1.695	1.651	1.615	380	1.587	1.563	1.544
135	1.689	1.647	1.612	385	1.586	1.563	1.543
140	1.685	1.643	1.609	390	1.585	1.562	1.543
145	1.680	1.639	1.606	395	1.584	1.561	1.542
150	1.676	1.636	1.603	400	1.583	1.560	1.542
155	1.672	1.633	1.600	405	1.582	1.560	1.541
160	1.668	1.630	1.598	410	1.581	1.559	1.540
165	1.664	1.627	1.595	415	1.580	1.558	1.540
170	1.661	1.624	1.593	420	1.579	1.558	1.539
175	1.657	1.621	1.591	425	1.579	1.557	1.539
180	1.654	1.619	1.589	430	1.578	1.556	1.538
185	1.651	1.616	1.587	435	1.577	1.556	1.538
190	1.648	1.614	1.585	440	1.576	1.555	1.537
195	1.645	1.612	1.583	445	1.576	1.554	1.537
200	1.643	1.609	1.581	450	1.575	1.554	1.536
205	1.640	1.607	1.580	455	1.574	1.553	1.536
210	1.638	1.605	1.578	460	1.573	1.553	1.535
215	1.635	1.603	1.576	465	1.573	1.552	1.535
220	1.633	1.601	1.575	470	1.572	1.552	1.534
225	1.631	1.600	1.573	475	1.571	1.551	1.534
230	1.629	1.598	1.572	480	1.571	1.550	1.533
235	1.627	1.596	1.571	485	1.570	1.550	1.533
240	1.625	1.595	1.569	490	1.569	1.549	1.532
245	1.623	1.593	1.568	495	1.569	1.549	1.532
250	1.621	1.591	1.567	500	1.568	1.548	1.532
255	1.619	1.590	1.566	505	1.567	1.548	1.531

$n = 10(5)505$, α-risk $= 0.05$, 0.025, and 0.01. Using those tables, the practitioners may perform the capability testing without having to run the SAS computer programs. In practice, the sample data taken from a stable normal process is first collected. The sample mean and the sample standard deviation, $\overline{X}$ and S, are calculated. The estimator $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$) is then calculated, and compared with the critical value c_0 found from the table.

Table 4
Critical values c_0 for $C = 1.60$, $n = 10(5)505$ and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	3.074	2.700	2.430	260	1.783	1.752	1.725
15	2.652	2.410	2.228	265	1.781	1.750	1.724
20	2.450	2.265	2.122	270	1.779	1.749	1.723
25	2.327	2.175	2.056	275	1.778	1.747	1.722
30	2.244	2.112	2.009	280	1.776	1.746	1.721
35	2.183	2.066	1.973	285	1.774	1.744	1.719
40	2.135	2.030	1.946	290	1.773	1.743	1.718
45	2.098	2.001	1.923	295	1.771	1.742	1.717
50	2.066	1.977	1.904	300	1.769	1.741	1.716
55	2.040	1.956	1.889	305	1.768	1.739	1.715
60	2.018	1.939	1.875	310	1.767	1.738	1.714
65	1.998	1.924	1.863	315	1.765	1.737	1.713
70	1.981	1.910	1.852	320	1.764	1.736	1.712
75	1.966	1.899	1.843	325	1.762	1.735	1.712
80	1.953	1.888	1.835	330	1.761	1.734	1.711
85	1.941	1.878	1.827	335	1.760	1.733	1.710
90	1.930	1.870	1.820	340	1.758	1.732	1.709
95	1.920	1.862	1.814	345	1.757	1.731	1.708
100	1.910	1.854	1.808	350	1.756	1.730	1.707
105	1.902	1.847	1.803	355	1.755	1.729	1.707
110	1.894	1.841	1.798	360	1.754	1.728	1.706
115	1.887	1.835	1.793	365	1.753	1.727	1.705
120	1.880	1.830	1.789	370	1.752	1.726	1.704
125	1.874	1.825	1.784	375	1.750	1.725	1.704
130	1.868	1.820	1.781	380	1.749	1.724	1.703
135	1.862	1.816	1.777	385	1.748	1.723	1.702
140	1.857	1.811	1.774	390	1.747	1.722	1.701
145	1.852	1.807	1.770	395	1.746	1.722	1.701
150	1.847	1.804	1.767	400	1.745	1.721	1.700
155	1.843	1.800	1.764	405	1.744	1.720	1.700
160	1.839	1.797	1.762	410	1.743	1.719	1.699
165	1.834	1.793	1.759	415	1.743	1.719	1.698
170	1.831	1.790	1.757	420	1.742	1.718	1.698
175	1.827	1.787	1.754	425	1.741	1.717	1.697
180	1.824	1.785	1.752	430	1.740	1.716	1.696
185	1.820	1.782	1.750	435	1.739	1.716	1.696
190	1.817	1.779	1.748	440	1.738	1.715	1.695
195	1.814	1.777	1.746	445	1.737	1.714	1.695
200	1.811	1.774	1.744	450	1.737	1.714	1.694
205	1.808	1.772	1.742	455	1.736	1.713	1.694
210	1.805	1.770	1.740	460	1.735	1.712	1.693
215	1.803	1.768	1.738	465	1.734	1.712	1.693
220	1.800	1.766	1.737	470	1.733	1.711	1.692
225	1.798	1.764	1.735	475	1.733	1.710	1.692
230	1.796	1.762	1.734	480	1.732	1.710	1.691
235	1.793	1.760	1.732	485	1.731	1.709	1.691
240	1.791	1.758	1.731	490	1.731	1.709	1.690
245	1.789	1.756	1.729	495	1.730	1.708	1.690
250	1.787	1.755	1.728	500	1.729	1.708	1.689
255	1.785	1.753	1.727	505	1.729	1.707	1.689

A simple step-by-step procedure (Test Procedure I) based on the critical value, is presented bellow for in-plant applications. The practitioners can use the proposed procedure to determine whether or not their processes meet the preset capability requirements and run under the desired quality conditions. If the SAS computer program is used, then Test Procedure II based on the p-value may be applied.

3.1. Test Procedure I (based on critical value c_0)

Step 1: Determine the value of C, the desired quality condition, and the α-risk (normally set to 0.01, 0.025, or 0.05), the chance of incorrectly concluding a bad process (quality does not meet the capability requirement) as a good process (quality meets the preset capability requirement).

Step 2: Calculate the value of the estimator, $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$), from the sample.

Step 3: Check the appropriate Tables 2–4 and find the corresponding c_0 based on C, α, and n.

Step 4: Conclude that the process meets the capability requirement if $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$) is greater than c_0. Otherwise, we do not have sufficient information to conclude that the given process satisfies the capability requirement. In this case, we tend to believe that the process is incapable.

3.2. Test Procedure II (based on the p-value)

Step 1: Determine the value of C, the desired quality condition, and the α-risk (normally set to 0.01, 0.025, or 0.05), the chance of incorrectly concluding a bad process (quality does not meet the capability requirement) as a good process (quality meets the preset capability requirement).

Step 2: Calculate the value of the estimator, $\widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$), from the sample.

Step 3: Input C, n, and $w = \widetilde{C}_{PU}$ (or $\widetilde{C}_{PL}$), and execute the provided SAS program listed in Appendix A to find the corresponding p-value.

Step 4: Conclude that the process meets the capability requirement if the p-value is less than the chosen risk α. Otherwise, we do not have enough information to conclude that the given process satisfies the capability requirement. In this case, we tend to believe that the process is incapable.

4. The voltage level translator

The transistor–transistor logic (TTL) family has been the major family of bipolar digital ICs for over 30 years. TTL had been the leading IC family in the small-scale integration (SSI, with fewer than 12 gates per chip) and median-scale integration (MSI, with 12–99 gates per chip) categories up until the last 10 or so years. Since then the leading position has been challenged by the CMOS, the complementary metal–oxide semiconductor family, which has gradually displaced TTL from that position. CMOS belongs to the class of unipolar digital ICs, which uses fewer components in many high performance applications, one of the main advantages of CMOS over TTL. CMOS and TTL ICs dominate the field of SSL and MSI devices. Both the TTL and CMOS circuits have a dc power supply voltage connected to one of their pins, and ground to another. The power supply pin is labeled V_{CC} for the TTL circuit, and V_{DD} for the CMOS circuit. Many of the newer CMOS ICs that are designed to be compatible with TTL ICs also use the V_{CC} designation as their power pin. For TTL devices, V_{CC} is nominally 5 V. For CMOS ICs, V_{DD} can range from 3 to 18 V, although 5 V is most often used when CMOS ICs are used in the same circuit with TTL ICs. When TTL ICs are used with the high-voltage CMOS operating at $V_{DD} = 15$ V, then a voltage-level translator is necessary if TTL is to be driven directly from the high-voltage CMOS since the specifications of most TTL types are unable to handle an input voltage for more than 7 V before becoming damaged. The voltage-level translator converts the high-voltage input to a suitable voltage output 5 V that can be connected to the TTL.

The example investigated is taken from a microelectronics factory, located on the Science-Based Industrial Park in Taiwan, manufacturing the voltage level translators. This factory manufactures various types of the voltage-level translator. For a particular model of the voltage-level translator investigated, the upper specification limit, USL, for the out voltage level was set to 6.8 V. The capability requirement for this particular model of voltage-level translator was set to $C_{PU} = 1.25$. The collected sample data (a total of 120 observations) are displayed in Table 5.

Fig. 5 displays the histogram of the 120 observations. Fig. 6 displays the normal probability plot of the sample data. The sample data appears to be normal. Shapiro–Wilk test for normality check is also performed,

Table 5
Sample data of 120 voltage level translators

4.57	4.72	5.00	5.17	5.14	5.23
4.91	5.95	4.70	4.08	5.19	5.37
4.42	5.16	5.56	5.48	5.07	5.34
5.62	4.63	4.58	5.55	5.67	5.46
4.57	4.94	4.94	5.16	5.35	5.35
4.64	4.55	5.16	4.85	5.08	5.70
4.54	4.33	5.64	4.52	5.35	4.74
5.49	4.90	5.48	4.49	5.06	4.85
4.98	4.24	4.87	4.17	5.07	5.03
4.50	4.47	4.66	4.50	4.51	4.19
4.63	5.46	4.47	4.67	3.95	4.90
5.34	5.32	4.55	4.84	4.85	4.73
5.27	5.17	5.07	4.15	4.74	4.70
4.56	5.01	4.29	5.41	4.35	4.70
5.47	4.99	5.09	4.90	4.34	5.47
5.03	4.14	5.24	5.36	4.69	5.19
5.07	4.67	5.33	5.50	4.76	4.82
5.13	5.12	4.84	4.89	5.64	5.10
4.64	4.63	5.12	5.06	5.28	4.46
5.14	4.95	5.96	5.35	3.87	4.65

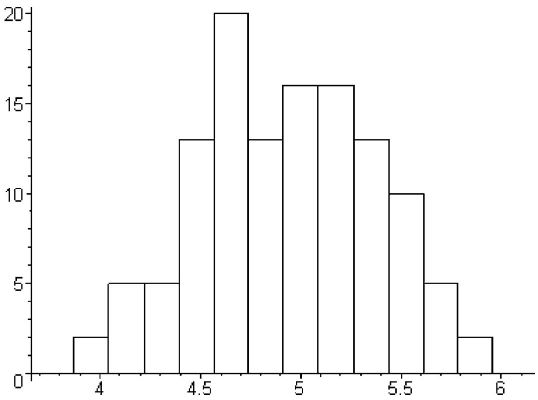

Fig. 5. Histogram of the sample data.

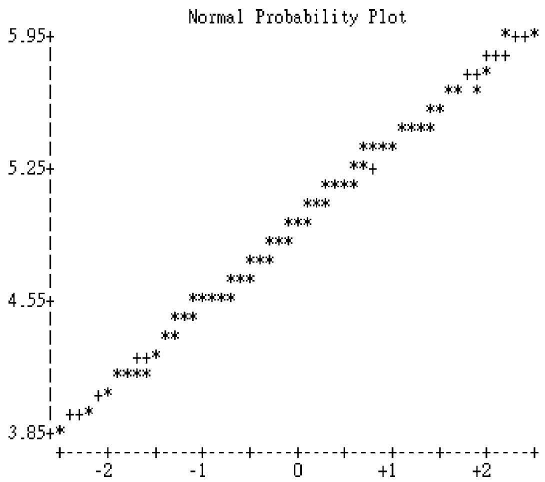

Fig. 6. The normal probability plot.

obtaining $W = 0.9922$. Thus, the sample data can be regarded as taken from a normal process.

The sample mean $\overline{X} = 4.94$ and sample standard deviation $S = 0.43$ are first calculated. For $n = 120$, we obtain the correction factor $b_{n-1} = 0.9937$, and calculate the value of the estimator $\widetilde{C}_{PU} = b_{n-1}(USL - \overline{X})/(3S) = 1.433$. Assume the α-risk is 0.05, the critical value is found to be $c_0 = 1.401$ from Table 2 based on $C = 1.25$, $\alpha = 0.05$, and $n = 120$. Since $\widetilde{C}_{PU} = 1.433$ is greater than the critical value $c_0 = 1.401$ in this case, it is therefore concluded with 95% confidence ($\alpha = 0.05$) that the voltage level translator manufacturing process satisfies the requirement '$C_{PU} > 1.25$'. It is noted that corresponding to the value of the estimator $w = \widetilde{C}_{PU} = 1.433$, sample size $n = 120$, and the capability requirement $C = 1.25$, the p-value is found to be 0.025 by executing the provided SAS program. Thus, the actual probability of incorrectly concluding an incapable process as a capable one is 2.5%.

Appendix A

```
/*****************************/;
/* SAS PROGRAM for p-Value    */;
/*****************************/;
OPTIONS REPLACE PAGESIZE = 78;
OPTIONS LINESIZE = 78 NODATE;
DATA VoLeTranlator;
/*****************************/;
/*Input quality requirement C */
/*Input Sample size n         */
/*Input w = Estimated CPU      */
/*****************************/;
C = 1.25; n = 120; w = 1.433;
F = n-1; ND = 3*SQRT(n) *C;
/*****************************/;
/*Calculate bf                */
/*Find the P-value            */
/*****************************/;
DN = SQRT((n − 2)/2) * (1 − 1/(4 * (n − 2))+
1/(32 * (n − 2) * *2) + 5/(128 * (n − 2) * *3));
BF = SQRT(2/(n − 1)) * DN;
X = 3 * SQRT(n) * w/BF;
PV = 1 − PROBT(X, F, ND);
OUTPUT;
FORMAT C 4.2 PV 5.3;
PROC PRINT DATA = VoLeTranlator;
VAR C n w PV;
RUN;
```

The output is:

The SAS System

```
Obs    C      n     w      PV
1      1.25   120   1.433  0.025
```

Appendix B

```
*****************************/;
/* SAS PROGRAM for the      */;
/* Critical Value co       */;
/*****************************/;
OPTIONS REPLACE PAGESIZE = 78;
OPTIONS LINESIZE = 78 NODATE;
DATA VoLeTranlator;
/*****************************/;
/*Input capability Requirement C */
/*Input Risk Alpha            */
/*Input Sample Size n         */
/******************************/;
C = 1.25;
Alpha = 0.05;
n = 120;
```

```
F = n - 1;
ND = 3 * SQRT(n) * C;
/******************************/;
/*Calculate bf               */
/*Find Critical value c0     */
/******************************/;
DN = SQRT((n - 2)/2) * (1 - 1/(4 * (n - 2)) +
1/(32 * (n - 2) * *2) + 5/(128 * (n - 2) * *3));
BF = SQRT(2/(n - 1)) * DN;
CO = BF/(3 * SQRT(n) * TINV(1 - Alpha, F, ND);
FORMAT CO 5.3;
PROC PRINT DATA = VoLeTranlator;
VAR C Alpha n CO;
RUN;
```

The output is:

```
The SAS System

Obs    C      Alpha    n      CO
1      1.25   0.05     120    1.401
```

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